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Beyond the $c = 1$ Barrier in Two-Dimensional Quantum Gravity

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Abstract

We introduce a simple model of *touching* random surfaces, by adding a chemical potential ρ for "minimal necks", and study this model numerically coupled to a Gaussian model in d–dimensions (for central charge $c = d = 0$, 1 and 2). For $c \leq 1$, this model has a phase transition to branched polymers, for sufficiently large ρ. For $c = 2$, however, the extensive simulations indicate that this transition is replaced by a cross-over behavior on finite lattices — the model is always in the branched polymer phase. This supports recent speculations that, in $2d$–gravity, the behavior observe in simulations for $c \leq 1$, is dominated by finite size effects, which are exponentially enhanced as $c \to 1^+$.

1 Introduction

When conformally invariant matter $\mathcal{S}_M(X)$ is coupled to two-dimensional quantum gravity:

$$\mathcal{Z}(\mu) \;=\; \int \mathcal{D}g\,\mathcal{D}X\;\mathrm{e}^{\,-\mu\,\int \mathrm{d}^2\xi\sqrt{|g|}\,-\,\mathcal{S}_m(X;g)}\;, \tag{1}$$

this breaks down when the matter *central charge* c becomes larger than *one*. We get unphysical *complex* critical exponents, such as the *string susceptibility exponent* γ_s: $\mathcal{Z}(\mu) \;\sim\; (\mu_c - \mu)^{2-\gamma_s}$; given by the *KPZ*–scaling relation:

$$\gamma_s \;=\; \frac{1}{12}\left(c - 1 - \sqrt{(c-25)(c-1)}\right) \tag{2}$$

Hence, predictions of continuum theories become meaningless for $c > 1$. This puzzle, which is related to the occurrence of *tachyons* in bosonic string theories in $d > 2$, still remains a challenging problem in $2d$–gravity.

Discretized models of $2d$–gravity are, on the other hand, well defined for $c > 1$, and suitable for studying this problem. Simplicial gravity, alias dynamical triangulations, is a discretization of quantum gravity with integrations over metrics replaced by all possible gluing's of *simplices* into piecewise linear manifolds T:

$$\mathcal{Z} \;=\; \sum_{A} \mathrm{e}^{\,-\mu A} \sum_{T \in \mathcal{T}(A)} \mathcal{Z}_M \;. \tag{3}$$

A is the area of the surface, $\mathcal{Z}_M$ the (discretized) matter partition function; for example, a d–dimensional Gaussian model (bosonic string theory with $c = d$):

$$\mathcal{Z}_M \;=\; \int \mathrm{d}^d x \; \delta(x_{cm}) \; \mathrm{e}^{\,-\sum_{<ij>} (\vec{x}_i - \vec{x}_j)^2} \;, \tag{4}$$

and $\mathcal{T}$ is an appropriate *class* of triangulations; different classes amount to different discretizations of the manifolds, but should yield the same continuum theory. Commonly used are *combinatorial* ($\mathcal{T}_C$) and *degenerate* ($\mathcal{T}_D$) triangulations.

Models of dynamical triangulations have been studied extensively, both as *matrix models* (for $c \leq 1$) and using numerical simulations. What have we learned so far:

- For $c \leq 1$ the models are well understood; γ_s agrees with the *KPZ*–scaling and the (internal) fractal dimension of the triangulations ($A(r) \sim r^{d_H}$) is $d_H \approx 4$ (still somewhat controversial).
- For $c \gtrsim 5$ the dominant triangulations are *branched polymers* (bubbles glued together in a tree-like structure) with $\gamma_s = 1/2$ and $d_H = 2$.
- But, for $1 < c \lesssim 5$ the situation is still unclear. Numerical simulations indicate a *smooth* cross-over to the branched polymer phase as c increases.

Is this due to very big finite-size effects [1], or is there a different critical behavior for $1 < c \lesssim 5$?

2 Touching random surfaces

A conjecture for the observed $c > 1$ behavior, was put forward in [2]: "For $c > 1$ the dynamical triangulation model is *always* in a branched polymer phase. But finite size effects are *exponentially enhanced* as $c \to 1^+$, due to the influence of the $c = 1$ fixed point (which becomes *complex* for $c > 1$)."

This is based on a large–N renormalization group analysis of a matrix model including "touching" interactions:

$$\mathcal{Z} \;=\; \int \mathrm{d}M \; \mathrm{e}^{\,-N \,\mathrm{tr}(M^2 + gM^4) \,-\, x\,(\mathrm{tr}(M^2))^2} \;. \tag{5}$$

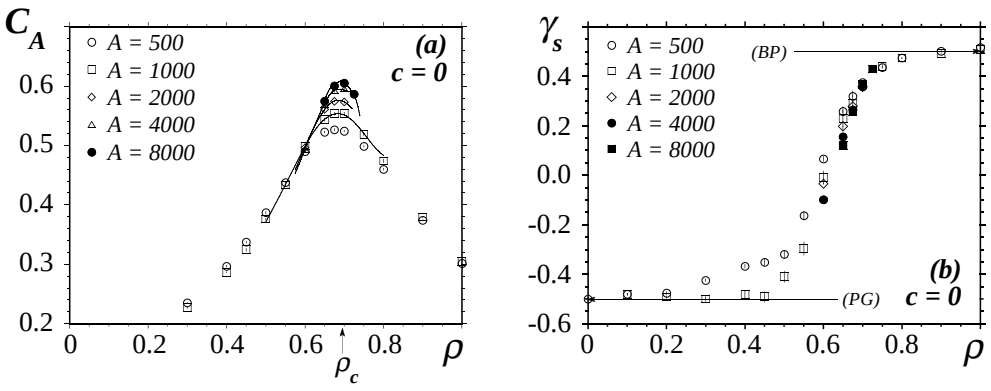

Figure 1: C_A and γ_s for $c = 0$.

For $c \leq 1$ this model has a transition to branched polymers at a critical value of the *touching* coupling x [3]. For $c > 1$, however, this fixed point moves into the complex plane; but it still influences the *RG*–flow's when c is not too big.

How do we verify this conjecture? We introduce a simple model of touching random surfaces, adding a *chemical potential* ρ for *minimal necks* n_m on the surface. As we work with degenerate triangulations, a minimal neck is a vertex connected to itself *via* a link (a *tadpole* in the dual graph). The (fixed area) partition function is:

$$\mathcal{Z}_A(\rho) \;=\; \sum_{T \in \mathcal{T}_D} \mathrm{e}^{\,\rho n_m}\; \mathcal{Z}_M \;. \tag{6}$$

We have simulated this model for $c \leq 2$, using 0, 1 and 2 Gaussian models, on surfaces up to 8000 triangles. Our goal is to verify the existence of a transition to branched polymers for $c \leq 1$, and to see if this transition still exists for $c > 1$. Or, alternatively, is it replaced by *cross-over* behavior on finite lattices.

To study the phase structure of we measure the second derivative of the free energy: $C_A \;=\; A^{-1}\;\partial^2 \log \mathcal{Z}_A / \partial \rho^2$, and the string susceptibility exponent γ_s. The latter is obtained from the distribution of *baby universes* on the surface, using the large–A behavior of the partition function: $\mathcal{Z}_A \;\approx\; \mathrm{e}^{\mu_c A} A^{\gamma_s - 3}$. For $c = 1$ this behavior is modified by logarithmic corrections, $\mathcal{Z}_A \;\approx\; \mathrm{e}^{\mu_c A} A^{\gamma_s - 3} \log^{\alpha} A$, — including them is essential to extract the correct γ_s numerically [4].

3 Results

For $c = 0$ (pure gravity) we see a clear signal of a phase transition. There is a peak in C_A, which gets sharper as A increases, but does not diverge (Fig. 1*a*). Finite size scaling of the peak ($C_A \sim c_0 + c_1 A^{\alpha/\nu d_H}$) gives: $\rho_c = 0.695(5)$ and $\alpha = -1.07(11)$, assuming hyper-scaling is valid ($\alpha = 2 - \nu d_H$). (Note that this ν is related to the

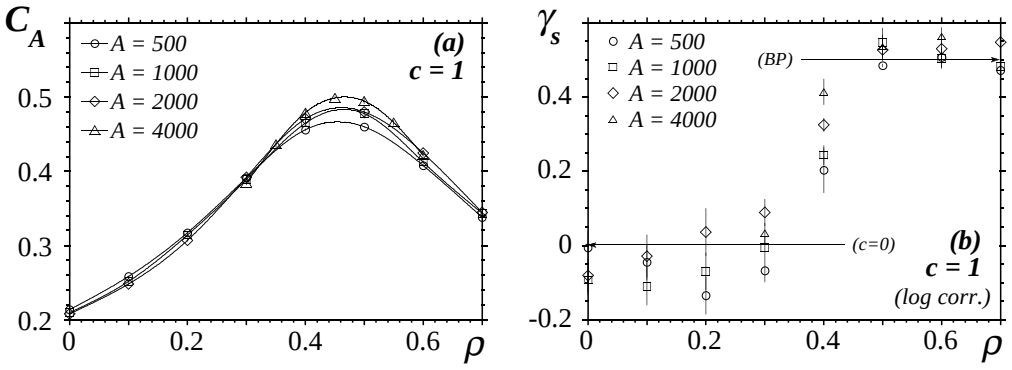

Figure 2: C_A and γ_s for $c = 1$.

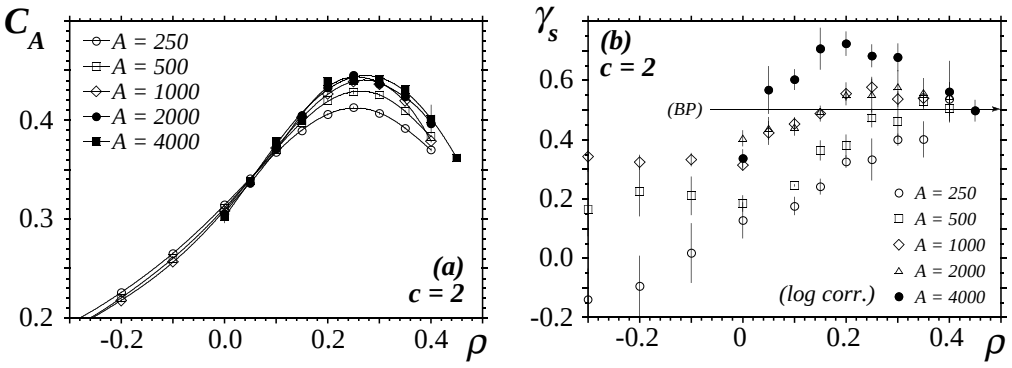

Figure 3: C_A and γ_s for $c = 2$.

touching interaction; hence $\nu \neq 1/d_H$). At the same value of ρ_c there is a sharp transition in γ_s from its pure gravity value, $\gamma_s(PG) = -1/2$, to branched polymers, $\gamma_s(BP) = 1/2$ (Fig. 1*b*).

We observe a similar behavior for $c = 1$ (Figs. 2*a* and *b*): a non-divergent peak in C_A, with $\rho_c = 0.45(1)$ and $\alpha = -0.8(2)$, accompanied by a transition to branched polymers in γ_s. In this case, γ_s is extracted using logarithmic corrections, with α as a free parameter. Below ρ_c, $\alpha \approx -1$, whereas $\alpha \approx 0$ for branched polymers.

For $c > 1$, on the other hand, the behavior is different. We still observe a peak in C_A (Fig. 3*a*), but it saturates faster than for $c \leq 1$. In fact, $\alpha/\nu d_H < -1$, which implies, if this is a phase transition, that hyper-scaling is violated. And, more important, there is *no* indication of a phase transition in γ_s, only a smooth cross-over to branched polymers, which seems to disappear as $A \to \infty$ (Fig. 3*b*). This is

independent of the corrections included in extracting γ_s. This behavior is, in our opinion, not compatible with the existence of a phase transition, and we conclude that there is only a branched polymer phase. This strongly supports the conjecture in [2] about the nature of the $c = 1$ "barrier".

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